$$\int \frac{\boldsymbol{x^2 - x - 5}}{\boldsymbol{x^3 + x^2 - 2}} \boldsymbol{dx}$$

Para factorizar $x^3 + x^2 - 2$, se hace una división sintética considerando como primer factor $x - r$, con $r = 1$

1|	1	1	0	− 2
		1	2	2
	1	2	2	0

factores $(x - 1)$ y $(x^2 + 2x + 2)$, de manera que:

$$\int \frac{x^2 - x - 5}{x^3 + x^2 - 2} dx = \int \frac{Ax + B}{x^2 + 2x + 2} dx + \int \frac{C}{x - 1}$$

$$\frac{x^2 - x - 5}{(x-1)(x^2 + 2x + 2)} = \frac{Ax + B}{x^2 + 2x + 2} + \frac{C}{x - 1}$$

$$x^2 - x - 5 = (Ax + B)(x - 1) + C(x^2 + 2x + 2)$$

Si $\boldsymbol{x = 1}$

$$1^2 - 1 - 5 = (A(1) + B)(1 - 1) + C(1^2 + 2(1) + 2)$$

$$-5 = C(1 + 2 + 2) = 5C \qquad \therefore \qquad C = \frac{-5}{5} = -1 \qquad \boldsymbol{C = -1}$$

Sustituyendo C en:

$$x^2 - x - 5 = (Ax + B)(x - 1) + (-1)(x^2 + 2x + 2)$$

$$x^2 - x - 5 = Ax^2 - Ax + Bx - B - x^2 - 2x - 2$$

$$x^2 - x - 5 = (A - 1)x^2 + (-A + B - 2)x + (-B - 2)$$

Por analogía:

$$1 = A - 1 \qquad A = 1 + 1 = 2 \qquad \therefore \qquad \boldsymbol{A = 2}$$

$-1 = -A + B - 2 \qquad -1 = -2 + B - 2 \qquad B = -1 + 4 \quad \therefore \quad \boldsymbol{B = 3}$

$-5 = -B - 2$

Sustituyendo estos valores en la integral se tiene:

$$\int \frac{x^2 - x - 5}{x^3 + x^2 - 2} dx = \int \frac{Ax + B}{x^2 + 2x + 2} dx + \int \frac{C}{x - 1}$$

$$= \int \frac{2x+3}{x^2+2x+2} dx + \int \frac{-1}{x-1} dx$$

Donde $\int \frac{2x+3}{x^2+2x+2} dx \;\; se\ puede\ reescribir\ como$

$$= \int \frac{2x + 2 + 1}{x^2 + 2x + 2} = \int \frac{2x + 2}{x^2 + 2x + 2} + \int \frac{1}{x^2 + 2x + 2}$$

De manera que

$$\int \frac{x^2 - x - 5}{x^3 + x^2 - 2} dx = \int \frac{2x + 2}{x^2 + 2x + 2} + \int \frac{1}{x^2 + 2x + 2} + \int \frac{-1}{x - 1} dx$$

I II III

Resolviendo para la primera integral, I $\quad \int \frac{du}{u} = \boldsymbol{ln|u| + c}$

Si u = x^2+ 2x +2 entonces du = (2x +2)dx y se tiene que

$$\int \frac{2x + 2}{x^2 + 2x + 2} = Ln\,|x^2 + 2x + 2| + c$$

Resolviendo para la segunda integral, II $\quad \int \frac{du}{a^2+u^2} = \frac{1}{a} \boldsymbol{tan^{-1}\left(\frac{u}{a}\right) + c}$

$$\int \frac{1}{x^2+2x+2} = \int \frac{1}{(x+1)^2+1} = \frac{1}{1} tan^{-1}\left(\frac{x+1}{1}\right) + c = tan^{-1}(x + 1)$$

u = $(x + 1)^2$ a^2 = 1

Resolviendo para la tercera integral, III $\int \frac{du}{u} = ln|u| + c$

$$\int \frac{-1}{x-1} dx = -Ln|x-1| + c$$

Por lo que

$$\int \frac{x^2 - x - 5}{x^3 + x^2 - 2} dx = Ln\,|x^2 + 2x + 2| + \, tan^{-1}(x+1) - Ln|x-1| + C$$