$$\int \frac{\boldsymbol{x^4 + x^2 + 16x - 12}}{\boldsymbol{x^3(x-2)}^2}\boldsymbol{dx}$$

$(\boldsymbol{x}) = \frac{\boldsymbol{P(x)}}{\boldsymbol{Q(x)}}$: H(x) es un polinomio propio, ya que P(x) es de grado 4 y Q(x) es de grado 5.

Q(x) está formado por un factor lineal x que se repite 3 veces, x^3, y un factor lineal (x-2) que se repite 2 veces, $(x-2)^2$, de manera que.

$$\int \frac{x^4 + x^2 + 16x - 12}{x^3(x-2)^2}dx = \int \frac{A}{x}dx + \int \frac{B}{x^2}dx + \int \frac{C}{x^3}dx + \int \frac{D}{x-2}dx + \int \frac{E}{(x-2)^2}dx$$

$$\frac{x^4 + x^2 + 16x - 12}{x^3(x-2)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2} + \frac{E}{(x-2)^2}$$

$$x^4 + x^2 + 16x - 12 = Ax^2(x-2)^2 + Bx(x-2)^2 + C(x-2)^2 + Dx^3(x-2) + Ex^3$$

$\boldsymbol{Si\ x = 0}$

$$0^4 + 0^2 + 16(0) - 12 = A(0)^2(0-2)^2 + B(0)(0-2)^2 + C(0-2)^2 + D(0)^3(0-2) + E(0)^3$$

$$-12 = C(-2)^2 = C(4) \qquad C = \frac{-12}{4} = -3 \qquad \therefore \qquad \boldsymbol{C = -3}$$

$Si\ \boldsymbol{x = 2}$

$$2^4 + 2^2 + 16(2) - 12 = A(2^2)(2-2)^2 + B(2)(2-2)^2 + C(2-2)^2 + D(2)^3(2-2) + E(2)^3$$

$$16 + 4 + 32 - 12 = E(2)^3$$

$$40 = E(8) \qquad E = \frac{40}{8} \qquad \therefore \qquad \boldsymbol{E = 5}$$

Desarrollando los términos elevados al cuadrado:

$$x^4 + x^2 + 16x - 12 = Ax^2(x^2 - 4x + 4) + Bx(x^2 - 4x + 4) + C(x^2 - 4x + 4) + Dx^3(x-2) + Ex^3$$

$$= Ax^4 - 4Ax^3 + 4Ax^2 + Bx^3 - 4Bx^2 + 4Bx + Cx^2 - 4Cx + 4C + Dx^4 - 2Dx^3 + Ex^3$$

Sustituyendo los valores de C = -3; E = 5 y agrupando términos semejantes:

$$= (A + D)x^4 + (-4A + B - 2D + E)x^3 + (4A - 4B + C)x^2 + (4B - 4C)x - 4C$$

$$x^4 + x^2 + 16x - 12 = (A + D)x^4 + (-4A + B - 2D + 5)x^3 + (4A - 4B - 3)x^2 + (4B + 12)x + 12$$

Por analogía:

1 = A + D $(para\ x^4)$
0 = - 4A + B - 2D + 5 $(para\ x^3)$
$1 = 4A - 4B - 3$ $(para\ x^2)$
$16 = 4B + 12$ $(para\ x)$

De $16 = 4B + 12$ $\quad 16 - 12 = 4B$ $\quad 4 = 4B$ $\quad \therefore \quad \boldsymbol{B = 1}$

De $1 = 4A - 4B - 3$ $\quad 1 = 4A - 4(1) - 3$
$1 = 4A - 4 - 3$
$1 = 4A - 7$ $\quad 4A = 1 + 7$ $\quad A = \frac{8}{4}$ $\quad \therefore \quad \boldsymbol{A = 2}$

De $1 = A + D$ $\quad 1 = 2 + D$ $\quad 1 - 2 = D$ $\quad \therefore \quad \boldsymbol{D = -1}$

$\therefore \quad A = 2, \quad B = 1, \quad C = -3, \quad D = -1 \quad y \quad E = 5$

Entonces: $\quad \boldsymbol{\int \frac{du}{u} = ln|u| + c} \qquad \boldsymbol{\int u^n du = \frac{u^{n+1}}{n+1} + c}$

$$\int \frac{x^4 + x^2 + 16x - 12}{x^3(x-2)^2}dx = \int \frac{A}{x}dx + \int \frac{B}{x^2}dx + \int \frac{C}{x^3}dx + \int \frac{D}{x-2}dx + \int \frac{E}{(x-2)^2}dx$$

u=x, du=dx; u=x, du=dx, n=2; u=x, du=dx, n=3;

u=x-2, du=dx; u=(x-2), du=dx, n=2

$$= \int \frac{2}{x}dx + \int \frac{1}{x^2}dx + \int \frac{-3}{x^3}dx + \int \frac{-1}{x-2}dx + \int \frac{5}{(x-2)^2}dx$$

$$= 2Ln|x| - x^{-1} + \frac{3}{2}x^{-2} - Ln|x-2| - 5(x-2)^{-1} + C$$

$$= 2Ln|x| - Ln|x-2| - \frac{1}{x} + \frac{3}{2x^2} - \frac{5}{x-2} + C$$