$$\int \frac{\boldsymbol{x-1}}{\boldsymbol{x^3 - x^2 - 2x}} \boldsymbol{dx}$$

$\boldsymbol{H(x)} = \frac{\boldsymbol{P(x)}}{\boldsymbol{Q(x)}}$: H(x) es un polinomio propio, ya que P(x) es de grado 1 y Q(x) es de grado 3.

Q(x) se puede factorizar como**:** $\boldsymbol{x^3 - x^2 - 2x = x(x^2 - x - 2) = x(x-2)(x+1)}$ tres factores lineales que no se repiten, de manera que.

$$\int \frac{\boldsymbol{x-1}}{\boldsymbol{x^3-x^2-2x}}\boldsymbol{dx} = \int \frac{\boldsymbol{A}}{\boldsymbol{x}}\boldsymbol{dx} + \int \frac{\boldsymbol{B}}{\boldsymbol{x-2}}\boldsymbol{dx} + \int \frac{\boldsymbol{C}}{\boldsymbol{x+1}}\boldsymbol{dx}\ \boldsymbol{,}$$

hay que obtener los valores de A, B y C, por lo que

$\frac{x-1}{x^3-x^2-2x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$, multiplicando por Q(x) en su forma factorizada

$x - 1 = A(x-2)(x+1) + Bx(x+1) + Cx(x-2)$

Resolviendo para los valores de x = 0, 2 y -1.

Si $\boldsymbol{x = 0}$

$0 - 1 = A(0-2)(0+1) + B(0)(0+1) + C(0)(0-2)$

$-1 = A(-2)(1) = -2A \qquad A = \frac{-1}{-2} = \frac{1}{2} \qquad \boldsymbol{A = \frac{1}{2}}$

Si $\boldsymbol{x = 2}$

$2 - 1 = A(2-2)(2+1) + B(2)(2+1) + C(2)(2-2)$

$1 = B(2)(3) = 6B \qquad \therefore \qquad \boldsymbol{B = \frac{1}{6}}$

Si $\boldsymbol{x = -1}$

$-1 - 1 = A(-1-2)(-1+1) + B(-1)(-1+1) + C(-1)(-1-2)$

$$-2 = C(-1)(-3) = 3C \quad \therefore \quad C = \frac{-2}{3} \qquad \boldsymbol{C = -\frac{2}{3}}$$

$$\int \frac{x-1}{x^3-x^2-2x}dx = \int \frac{A}{x}dx + \int \frac{B}{x-2}dx + \int \frac{C}{x+1}dx \qquad \int \frac{du}{u} = \boldsymbol{ln|u| + c}$$

u=x du=dx u=x-2 du=dx u=x+1 du=dx

$$= \int \frac{\frac{1}{2}}{x}dx + \int \frac{\frac{1}{6}}{x-2}dx + \int \frac{-\frac{2}{3}}{x+1}dx$$

$$= \frac{1}{2}\int \frac{dx}{x} + \frac{1}{6}\int \frac{dx}{x-2} - \frac{2}{3}\int \frac{dx}{x+1}$$

$$= \frac{1}{2}Ln|x| + \frac{1}{6}Ln|x-2| - \frac{2}{3}Ln|x+1| + C$$

Aplicando propiedades de los logaritmos

$$\int \frac{x-1}{x^3-x^2-2x}dx = Ln\left|x^{\frac{1}{2}}\right| + Ln\left|(x-2)^{\frac{1}{6}}\right| - Ln\left|(x+1)^{\frac{2}{3}}\right| + C$$

$$\int \frac{x-1}{x^3-x^2-2x}dx = Ln\left|\frac{\left(x^{\frac{1}{2}}\right)(x-2)^{\frac{1}{6}}}{(x+1)^{\frac{2}{3}}}\right| + C$$