Ejemplos del Segundo Teorema Fundamental del Cálculo

Integrales definidas

1.- $\int_2^4 x^3\, dx$

Al ser una integral definida se aplica el segundo teorema fundamental del cálculo $\int_a^b f(x)dx = F(b) - F(a)$, donde la primitiva se obtiene con $\int u^n du = \frac{u^{n+1}}{n+1} + c$ y entonces ésta se calcula en los límites de integración.

$$\int_2^4 x^3\, dx = \left[\frac{x^4}{4}\right]_2^4 = \frac{4^4}{4} - \frac{2^4}{4} = \frac{256}{4} - \frac{16}{4} = 64 - 4 = 60\, u^2$$

F(b) – F(a)

2.- $\int_0^2 x^3\, dx = \left[\frac{x^4}{4}\right]_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = \frac{16}{4} - 0 = 4 - 0 = 4\, u^2$,

F(b) – F(a)

con $\int u^n du = \frac{u^{n+1}}{n+1} + c$

u = x; du = dx y n = 3

3.- $\int_{-1}^{2}(3x^2-2x+3)\,dx=\int_{-1}^{2}3x^2\,dx-\int_{-1}^{2}2x\,dx+\int_{-1}^{2}3\,dx$;

$\int u^n du=\frac{u^{n+1}}{n+1}+c$

k = 3; u = x; du = dx y n = 2 k = -2; u = x; du = dx y n = 1

$$=3\int_{-1}^{2}x^2\,dx-2\int_{-1}^{2}x\,dx+3\int_{-1}^{2}dx$$

$$=\left[3\frac{x^3}{3}\right]_{-1}^{2}-\left[2\frac{x^2}{2}\right]_{-1}^{2}+[3x]_{-1}^{2}=[x^3]_{-1}^{2}-[x^2]_{-1}^{2}+[3x]_{-1}^{2}$$

$$=[2^3-(-1)^3]-[2^2-(-1)^2]+[3(2)-3(-1)]$$

F(b) – F(a) F(b) – F(a) F(b) – F(a)

$$\int_{-1}^{2}(3x^2-2x+3)\,dx=[8-(-1)]-[4-1]+[6+3]=9-3+9=15\,u^2$$

4.- $\int_{1}^{4}\frac{1}{w^2}dw=\int_{1}^{4}w^{-2}\,dw=\left[\frac{w^{-2+1}}{-2+1}\right]_{1}^{4}=\left[\frac{w^{-1}}{-1}\right]_{1}^{4}=\left[-\frac{1}{w}\right]_{1}^{4}=\left[-\frac{1}{4}-(-\frac{1}{1})\right]=\frac{3}{4}u^2$

$\int u^n du=\frac{u^{n+1}}{n+1}+c$; con u = w; du = dw y n = -2 F(b) – F(a)

5.- $\int_{0}^{4}\sqrt{t}\,dt=\int_{0}^{4}t^{\frac{1}{2}}\,dt=\left[\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_{0}^{4}=\left[\frac{2}{3}t^{\frac{3}{2}}\right]_{0}^{4}=\left[\frac{2}{3}\sqrt{t^3}\right]_{0}^{4}=\left[\frac{2}{3}\sqrt{4^3}-\frac{2}{3}\sqrt{0^3}\right]=\frac{16}{3}-0=\frac{16}{3}u^2$

$\int u^n du=\frac{u^{n+1}}{n+1}+c$; con u = t; du = dt y $n=\frac{1}{2}$

F(b) – F(a)