Ejercicios.

Encontrar el valor de la integral de **f (x) = 4 − 2x en [a, b] = [0, 2]**, usando:

.) Rectángulos inscritos
..) Rectángulos circunscritos
…) Sumas de Riemann con valores medios de x

Solución con rectángulos inscritos

Si: **f(x) = 4 – 2x;** $\mathbf{x_i} = a + \frac{2i}{n} = 0 + \frac{2i}{n} = \frac{2i}{n}$; $\Delta\mathbf{x} = \frac{2-0}{n} = \frac{2}{n}$. Por lo que:

$$A_i = f(x_i)\Delta x = \left(4 - 2\left(\frac{2i}{n}\right)\right)\Delta x = \left(4 - \frac{4i}{n}\right)\frac{2}{n} = \frac{8}{n} - \frac{8}{n^2}i$$

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n}\left(\frac{8}{n} - \frac{8}{n^2}i\right) = \sum_{i=1}^{n}\frac{8}{n} - \sum_{i=1}^{n}\frac{8}{n^2}i$$

$$\sum_{i=1}^{n} A_i = (n)\frac{8}{n} - \frac{8}{n^2}(0 + 2 + 3 + ... + n - 1)$$

$$\sum_{i=1}^{n} A_i = (n)\frac{8}{n} - \frac{8}{n^2}\left(\frac{(n-1)(n)}{2}\right) = 8 - 4\left(\frac{n^2 - n}{n^2}\right) = 8 - 4\left(1 - \frac{1}{n}\right)$$

$$\therefore\ A(R) = \lim_{n\to\infty}\left(8 - 4 + \frac{1}{n}\right) = 8 - 4 + 0 = 4\,u^2$$

Solución con rectángulos circunscritos

Si: **f(x) = 4 – 2x;** $\mathbf{x_i} = a + \frac{2i}{n} = 0 + \frac{2i}{n} = \frac{2i}{n}$; $\Delta\mathbf{x} = \frac{2-0}{n} = \frac{2}{n}$. Por lo que:

$$A_i = f(x_i)\Delta x = \left(4 - 2\left(\frac{2i}{n}\right)\right)\Delta x = \left(4 - \frac{4i}{n}\right)\frac{2}{n} = \frac{8}{n} - \frac{8}{n^2}i$$

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n}\left(\frac{8}{n} - \frac{8}{n^2}i\right) = \sum_{i=1}^{n}\frac{8}{n} - \sum_{i=1}^{n}\frac{8}{n^2}i$$

$$\sum_{i=1}^{n} A_i = (n)\,\frac{8}{n} - \frac{8}{n^2}(1+2+3+\ldots+n)$$

$$\sum_{i=1}^{n} A_i = (n)\frac{8}{n} - \frac{8}{n^2}\left(\frac{(n)(n+1)}{2}\right) = 8 - 4\left(\frac{n^2+n}{n^2}\right) = 8 - 4\left(1+\frac{1}{n}\right)$$

$$\therefore\ A(R) = \lim_{n\to\infty}\left(8 - 4 - \frac{1}{n}\right) = 8 - 4 - 0 = 4\,u^2$$

Sumas de Riemann con valores medios de x

f (x) = 4 − 2x en [a, b] = [0, 2] Si se consideran n= 5 intervalos se tiene.

a = 0 < 0.4 < 0.8 < 1.2 < 1.6 < 2.0 = b

cuyo valores medios son:

0.2; 0.6; 1.0; 1.4 y 1.8

$$\Delta x = \frac{2-0}{n} = \frac{2}{5} = 0.4$$

Y su suma de Riemann queda como

$$\sum_{i=1}^{5} \mathbf{f}(\bar{\mathbf{x}})\Delta\mathbf{x}_i = [\mathbf{4}-\mathbf{2}(\mathbf{0.2})]\mathbf{0.4} + [\mathbf{4}-\mathbf{2}(\mathbf{0.6})]\mathbf{0.4} + [\mathbf{4}-\mathbf{2}(\mathbf{1.0})]\mathbf{0.4} + [\mathbf{4}-\mathbf{2}(\mathbf{1.4})]\mathbf{0.4} + [\mathbf{4}-\mathbf{2}(\mathbf{1.8})]\mathbf{0.4}$$

$$\sum_{i=1}^{5} \mathrm{f}(\bar{\mathrm{x}})\Delta\mathrm{x}_i = [3.6]0.4 + [2.8]0.4 + [2]0.4 + [1.2]0.4 + [0.4]0.4 = 4\,u^2$$