Ejemplo: Derivadas

1.- Dada $y = (3 - 2\,x)^5$, encontrar $\frac{dy}{dx}$

La fórmula a aplicar es

$\frac{d(u^n)}{dx} = nu^{n-1}\frac{d(u)}{dx}$, donde u = (3 - 2x) con n = 5

$$\frac{dy}{dx} = 5(3 - 2\,x)^{5-1}\frac{d}{dx}(3 - 2\,x); \qquad Recordando\ que\ \frac{d(u+v)}{dx} = \frac{d(u)}{dx} + \frac{d(v)}{dx}$$

$$\frac{dy}{dx} = 5(3 - 2\,x)^4\left[\frac{d}{dx}(3) - 2\frac{dx}{dx}\right]; \qquad \text{Si } \frac{d(c)}{dx} = 0; \quad \frac{d(cu)}{dx} = c\,\frac{d(u)}{dx}; \quad \frac{d(x)}{dx} = 1$$

$$\frac{dy}{dx} = 5(3 - 2\,x)^4\,(0 - 2(1)) = 5(-2)(3 - 2\,x)^4$$

$$\therefore \qquad \frac{dy}{dx} = -10(3 - 2\,x)^4 \text{ esto es } \frac{d[(3-2\,x)^5]}{dx} = -10(3 - 2\,x)^4$$

Ejemplo: Derivadas

2.- Si $y = (x^3 - 2\,x^2 + 3\,x - 1)^{11}$, encontrar $\frac{dy}{dx}$

Para aplicar $\frac{d(u^n)}{dx} = nu^{n-1}\frac{d(u)}{dx}$

$u = x^3 - 2x^2 + 3x - 1$

$n = 11$

De manera que,

$$\frac{dy}{dx} = 11(x^3 - 2\,x^2 + 3\,x - 1)^{11-1}\frac{d}{dx}(x^3 - 2\,x^2 + 3\,x - 1) \quad \frac{d(c)}{dx} = 0; \frac{d(cu)}{dx} = \text{c}\frac{d(u)}{dx}; \frac{d(x)}{dx} = 1$$

$$\frac{dy}{dx} = 11(x^3 - 2\,x^2 + 3\,x - 1)^{10}[\frac{d}{dx}x^3 - 2\frac{d}{dx}x^2 + 3\frac{dx}{dx} - \frac{d}{dx}(1)]$$

$$\frac{dy}{dx} = 11(x^3 - 2\,x^2 + 3\,x - 1)^{10}(3\,x^2 - 4\,x + 3 - 0)$$

$$\therefore \quad \frac{dy}{dx} = 11(3\,x^2 - 4\,x + 3)(x^3 - 2\,x^2 + 3\,x - 1)^{10}$$

Ejemplo: Derivadas

3. Si $y = cos(\frac{3x^2}{x+2})$, encontrar $\frac{dy}{dx}$

Considerando que $\frac{d(cos(u))}{dx} = -\text{sen}(u)\frac{d(u)}{dx}$, con $u = \frac{3x^2}{x+2}$, se tiene.

$$\frac{d}{dx}cos(\frac{3x^2}{x+2}) = -\,sen(\frac{3x^2}{x+2})\frac{d}{dx}(\frac{3x^2}{x+2})$$

De donde $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$

$$= -sen(\frac{3x^2}{x+2})(\frac{(x+2)\frac{d}{dx}3x^2 - 3x^2\frac{d}{dx}(x+2)}{(x+2)^2})$$

$$= -sen(\frac{3x^2}{x+2})(\frac{(x+2)[3\frac{d}{dx}x^2] - 3x^2[\frac{d}{dx}x + \frac{d}{dx}2]}{(x+2)^2})$$

$$= -sen(\frac{3x^2}{x+2})(\frac{(x+2)(6x) - 3x^2(1+0)}{(x+2)^2})$$

$$= -sen(\frac{3x^2}{x+2})(\frac{6x^2 + 12x - 3x^2}{(x+2)^2})$$

$$\therefore\ \frac{dy}{dx} = -(\frac{3x^2+12x}{(x+2)^2})sen(\frac{3x^2}{x+2})$$

Ejemplo: Derivadas

4.- Si $y = (\frac{sen(x)}{cos(2x)})^3$ sea $u = \frac{sen(x)}{cos(2x)}$ $con\ n = 3$, entonces $\frac{d(u^n)}{dx} = nu^{n-1}\frac{d(u)}{dx}$

$$\frac{d}{dx}(\frac{sen(x)}{cos(2x)})^3 = 3(\frac{sen(x)}{cos(2x)})^{3-1}\frac{d}{dx}(\frac{sen(x)}{cos(2x)}) \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

$$= 3(\frac{sen(x)}{cos(2x)})^2\left(\frac{cos(2x)\frac{d}{dx}(sen(x)) - sen(x)\frac{d}{dx}(cos(x))}{(cos(2x))^2}\right) \quad \frac{d(sen(u))}{dx} = \cos(u)\frac{d(u)}{dx},$$

$$= 3(\frac{sen(x)}{cos(2x)})^2(\frac{cos(2x)\,cos(x)\frac{dx}{dx} - sen(x)(-sen(2x))\frac{d}{dx}2x}{(cos(2x))^2}) \quad \frac{d(cos(u))}{dx} = -\text{sen}(u)\frac{d(u)}{dx}$$

$$= 3(\frac{sen(x)}{cos(2x)})^2(\frac{cos(2x)\,cos(x) - sen(x)\,(-sen(2x))[2\frac{d}{dx}x]}{(cos(2x))^2})$$

$$= 3(\frac{sen(x)}{cos(2x)})^2(\frac{cos(2x)\,cos(x) + 2\,sen(x)\,(sen(2x))}{(cos(2x))^2})$$

$$= 3(\frac{sen^2(x)}{cos^2(2x)})(\frac{cos(2x)\,cos(x) + 2\,sen(x)\,(sen(2x))}{cos^2(2x)})$$

$$\therefore \frac{dy}{dx} = 3(sen^2(x))(\frac{cos(2x)\,cos(x) + 2\,sen(x)(sen(2x))}{cos^4(2x)})$$